

day 42

HW/ 3.9/77-78

3.10/13-16, 31-32, 36-38

3.11/1-8due
Wed 10/30

3.9/85-87, 93

3.10/19-22, 43-44, 47, 51-53

3.11/9-12due Thurs-Fri
10/31

3.7/101

3.8/74-75

3.9/15-17

3.10/23-26, 54-55

59-61

3.11/13-18

4.1/1-5

all due Fri-Mon
11/4

day 42

3.9/69) a) The derivative of $\log_2 9 = \frac{1}{9 \log 2}$

$$\frac{d}{dx}(c) = 0$$

$$\frac{\ln 9}{\ln 2}$$

~~False~~

$$\log_2 9 = x$$

$$\ln(2^x = 9) \Rightarrow$$

$$x \ln 2 = \ln 9$$

$$x = \frac{\ln 9}{\ln 2}$$

b) $\ln(x+1) + \ln(x-1) = \ln(x^2-1)$
for all x .

False let $x=0$

c) $2^{x+1} = e^{2 \ln(x+1)}$

FALSE

$$2^{x+1} = e^{\ln 2^{x+1}}$$

$$= e^{(x+1) \ln 2}$$

d) $\frac{d}{dx}(\sqrt{2}^x) = x \sqrt{2}^{x-1}$

NOT just FALSE, but idiotically FALSE

Power Rule
 x is in the
base
exponent is
a constant

Exponential Fn
Rule
 x is in the
exponent
base is a
constant

Logarithmic Differentiation
Technique
 x is in the base
 x is in the
exponent

e) $\frac{d}{dx}(x^{\sqrt{2}}) = \sqrt{2} x^{\sqrt{2}-1}$

True!

Logarithms

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Actually

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

day 42

$$\frac{d}{dx}(\log_b|x|) = \frac{1}{(\ln b)x}$$

no surprise

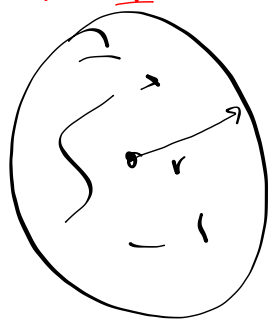
$$\frac{d}{dx}(\ln x) = \frac{d}{dx}(\log_e x) = \frac{1}{(\ln e)x} = \frac{1}{x}$$

3.11/example 1

day 42

An oil rig springs a leak in "calm seas" and the oil spreads in a circular patch around the rig. If the radius of the oil patch increases at 30m/hr, how fast is the area of the patch increasing when the patch has a radius of 100m?

step 1: picture



step 2: an equation that is true at every time.

$$A = \pi r^2$$

step 3: take the derivative w.r.t. t
 treat A like $A(t)$
 treat r like $r(t)$

$$\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2)$$

$$\frac{dA}{dt} = \pi \left(2r \frac{dr}{dt} \right) = 2\pi r \frac{dr}{dt}$$

step 4 Now, and ONLY NOW, substitute in

$$\frac{dA}{dt} = 2\pi(100)(30) = 6000\pi \frac{\text{m}^2}{\text{hr}}$$

Related Rates

day 42

3.9/52) determine whether the graf
of $y = x^{\sqrt{x}}$ has any horiz tangent lines

step 1 find derivative

$$\ln y = \ln(x^{\sqrt{x}}) = \sqrt{x} \ln x$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\sqrt{x} \ln x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}) \cdot \ln x + \sqrt{x} \frac{d}{dx}(\ln x)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{2} \frac{1}{\sqrt{x}} \cdot \ln x + \sqrt{x} \left(\frac{1}{x} \right)$$

$$\frac{dy}{dx} = y \left[\frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x} \right]$$

step 2 horizontal tangent means slope = 0

$$0 = y \left[\frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x} \right]$$

$$\left(\frac{\ln x}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} + \frac{2}{2} \cdot \frac{\sqrt{x}}{x} \right)$$

either ~~$y=0$~~ or

$$\sqrt{x} \ln x + 2\sqrt{x} = 0$$

[find common denominator;
set num = 0]

$$\sqrt{x} (\ln x + 2) = 0$$

$$\text{either } \sqrt{x} = 0 \quad \text{or } \ln x + 2 = 0$$

$$\text{or } \ln x + 2 = 0 \Rightarrow \ln x = -2 \Rightarrow x = e^{-2}$$

step 3 check in original